* Here is the fast way to "integrate by parts".

$$\int x^2 e^{2x}\,dx$$

Tabulate, differentiate the left and integrate the right...

x^2	e^{2x}	
$2x$	$\frac{1}{2}e^{2x}$	+
2	$\frac{1}{4}e^{2x}$	−
0	$\frac{1}{8}e^{2x}$	+

Multiply factors and add/subtract terms.

$$\boxed{\int x^2 e^{2x}\,dx = \frac{1}{2}x^2e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + C}$$

* By the traditional method, $u = x^2 \quad dv = e^{2x}dx$

$du = 2x\,dx \quad v = \frac{1}{2}e^{2x}$

$$\int u\,dv = uv - \int v\,du$$

$$= \frac{1}{2}x^2e^{2x} - \int (2x\,dx)\frac{1}{2}e^{2x}$$

$$= \frac{1}{2}x^2e^{2x} - \left(\int xe^{2x}dx\right)$$

$u = x \quad dv = e^{2x}dx$

$du = dx \quad v = \frac{1}{2}e^{2x}$

$$\left(\int u\,dv = uv - \int v\,du\right)$$

$$= \frac{1}{2}x^2e^{2x} - \left(\frac{1}{2}xe^{2x} - \int \frac{1}{2}e^{2x}dx\right) + C$$

$$= \frac{1}{2}x^2e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{2}\left(\frac{1}{2}e^{2x}\right) + C$$

$$= \frac{1}{2}x^2e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + C$$

* TAKE YOUR PICK. — WF