→ A REVIEW OF THE METHOD OF PARTIAL FRACTIONS

$\boxed{\text{Decompose } \frac{3x-1}{x^2-1} \text{ into Partial Fractions.}}$ (for use w/ Laplace Transform)

I. $\frac{3x-1}{x^2-1} = \frac{3x-1}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}$

II. Now clear the fractions,

$$3x-1 = A(x-1) + B(x+1)$$

III. use $x = -1$,

$$3(-1)-1 = A(-1-1) + 0$$

$$-4 = -2A$$

$$\underline{A = 2}$$

Now use $x = +1$

$$3(1)-1 = 0 + B(1+1)$$

$$2 = 2B$$

$$\underline{B = 1}$$

IV. So, $\frac{3x-1}{x^2-1} = \frac{2}{x+1} + \frac{1}{x-1}$

$\boxed{15.11}$

Decompose $\dfrac{1}{(s+1)(s^2+1)}$ into Partial Fractions.

I. We associate $\dfrac{A}{s+1}$ with the linear factor $(s+1)$

We associate $\dfrac{Bs+C}{s^2+1}$ to the quadratic factor s^2+1.

II. Then, $\dfrac{1}{(s+1)(s^2+1)} = \dfrac{A}{s+1} + \dfrac{Bs+C}{s^2+1}$

III. Clear the fractions.

$$1 = \frac{A(s+1)(s^2+1)}{(s+1)} + \frac{(Bs+C)(s+1)(s^2+1)}{(s^2+1)}$$

$$\begin{aligned} 1 &= A(s^2+1) + (Bs+C)(s+1) \\ &= As^2 + A + Bs^2 + Bs + Cs + C \\ &= s^2(A+B) + s(B+C) + (A+C) \end{aligned}$$

IV. Equate the coefficients of like powers of s.

$$\begin{cases} A+B=0 \\ B+C=0 \\ A+C=1 \end{cases}$$ and solve.

$A = -B$

$B = -C$

$A = 1 - C = -B = C$

$1 - C = C$

$1 - \frac{1}{2} = \frac{1}{2}$, $C = \frac{1}{2}$

$B = -\frac{1}{2}$

$A = \frac{1}{2}$

V. $\dfrac{1}{(s+1)(s^2+1)} = \dfrac{(\frac{1}{2})}{s+1} + \dfrac{-\frac{1}{2}s + \frac{1}{2}}{s^2+1}$

15.62 Decompose

$$\frac{1}{(s^2+1)(s^2+4s+8)}$$ into partial fractions.

I. Associate $\dfrac{As+B}{s^2+1}$ with (s^2+1), and

$\dfrac{Cs+D}{s^2+4s+8}$ with (s^2+4s+8)

II. $$\frac{1}{(s^2+1)(s^2+4s+8)} = \frac{As+B}{s^2+1} + \frac{Cs+D}{s^2+4s+8}$$

III. Clear the fractions.

$$\begin{aligned}
1 &= (As+B)(s^2+4s+8) + (Cs+D)(s^2+1) \\
&= As^3 + 4As^2 + 8As + Bs^2 + 4Bs + 8B \\
&\quad + Cs^3 + Ds^2 + Cs + D \\
&= As^3 + Cs^3 + 4As^2 + Bs^2 + Ds^2 \\
&\quad + 8As + 4Bs + Cs \\
&\quad + 8B + D \\
&= s^3(A+C) + s^2(4A+B+D) \\
&\quad + s(8A+4B+C) + (8B+D)
\end{aligned}$$

IV. Equate the coefficients of like powers of s,

$$\begin{cases} A+C=0 \\ 4A+B+D=0 \\ 8A+4B+C=0 \\ 8B+D=1 \end{cases} \qquad \begin{cases} A+0B+C+0D=0 \\ 4A+B+0C+D=0 \\ 8A+4B+C+0D=0 \\ 0A+8B+0C+D=1 \end{cases},$$

and solve.

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 4 & 1 & 0 & 1 & 0 \\ 8 & 4 & 1 & 0 & 0 \\ 0 & 8 & 0 & 1 & 1 \end{array}\right].$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 4 & 1 & 0 & 1 & 0 \\ 8 & 4 & 1 & 0 & 0 \\ 0 & 8 & 0 & 1 & 1 \end{array}\right]$$

... after many row operations (in Maple),

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -4/65 \\ 0 & 1 & 0 & 0 & 7/65 \\ 0 & 0 & 1 & 0 & 4/65 \\ 0 & 0 & 0 & 1 & 9/65 \end{array}\right]$$

V. So,

$$\frac{1}{(s^2+1)(s^2+4s+8)} = \frac{-\frac{4}{65}s + \frac{7}{65}}{s^2+1} + \frac{\frac{4}{65}s + \frac{9}{65}}{s^2+4s+8}.$$

$\boxed{15.14}$ Decompose $\frac{8}{s^3(s^2-s-2)}$ into partial fractions

$(s^2-s-2) = (s+1)(s-2)$
$s^3 = (s-0)^3$

I. We associate $\frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3}$ with $(s-0)^3$ &

$\frac{D}{s-2} + \frac{E}{s+1}$ with $(s+1)(s-2)$

II. Then $\frac{8}{s^3(s^2-s-2)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{D}{s-2} + \frac{E}{s+1}$

III. Clear out the fractions.

$$8 = As^2(s-2)(s+1)$$
$$+ Bs(s-2)(s+1)$$
$$+ C(s-2)(s+1)$$
$$+ Ds^3(s+1)$$
$$+ Es^3(s-2)$$

IV. Let $s = -1, 2,$ and 0 consecutively.

$s=-1$; $8 = A(-3)(0) + (-B)(-3)(0) + C(-3)(0) + D(0) - E(-3)$

$$8 = 3E$$
$$\underline{E = \frac{8}{3}}$$

$s=2$; $8 = 0+0+0+2^3D(3)+0$

$$8 = 24D$$
$$\underline{D = \frac{8}{24} = \frac{1}{3}}$$

$s=0$; $8 = 0+0+C(-2)(1)+0+0$

$$8 = -2C$$
$$\underline{C = -\frac{8}{2} = -4}$$

Now let $s = 1, -2$ and simplify.

We get, $A + B = -1$ and

$$2A - B = -8$$

where $A = -3$ & $B = 2$

V. So,

$$\frac{8}{s^3(s^2-s-2)} = -\frac{3}{s} + \frac{2}{s^2} - \frac{4}{s^3} + \frac{(1/3)}{s-2} + \frac{(8/3)}{s+1}$$