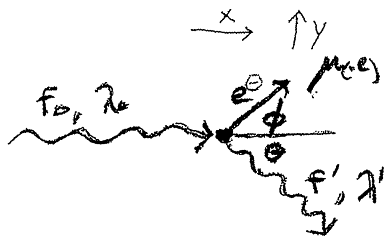
$[\mu_e] = \frac{m}{s}$



For a stationary electron,

Conservation of Energy

$$\frac{hc}{\lambda_0} = \frac{hc}{\lambda'} + (\gamma - 1) m_e c^2 \quad (1)$$

Conservation of Momentum

x-axis

$$\frac{h}{\lambda_0} = \frac{h}{\lambda'} \cos\theta + \gamma m_e u \cos\phi \quad (2)$$

y-axis

$$\frac{h}{\lambda'} \sin\theta = \gamma m_e u \sin\phi \quad (3)$$

$$\boxed{E = pc \qquad \frac{hc}{\lambda} = pc \qquad \frac{h}{\lambda} = p}$$

①

Isolate $\cos\phi$ and $\sin\phi$ from (2) and (3)

(4) $$\cos\phi = \frac{h}{\gamma m \mu}\left(\frac{1}{\lambda_0} - \frac{1}{\lambda'}\cos\theta\right)$$

(5) $$\sin\phi = \frac{h}{\gamma m \mu}\left(\frac{1}{\lambda'}\sin\theta\right)$$

$$\sin^2\phi + \cos^2\phi = 1 = \left(\frac{h}{\gamma m \mu}\right)^2\left[\left(\frac{1}{\lambda_0} - \frac{1}{\lambda'}\cos\theta\right)^2 + \left(\frac{1}{\lambda'}\sin\theta\right)^2\right]$$

$$1 = \left(\frac{h}{\gamma m \mu}\right)^2\left[\left(\frac{1}{\lambda_0^2} + \frac{1}{\lambda'^2}\cos^2\theta - \frac{2\cos\theta}{\lambda_0\lambda'}\right) + \left(\frac{1}{\lambda'^2}\sin^2\theta\right)\right]$$

$$= \left(\frac{h}{\gamma m \mu}\right)^2\left(\frac{1}{\lambda_0^2} + \frac{1}{\lambda'^2} - \frac{2\cos\theta}{\lambda_0\lambda'}\right)$$

$$1 = \left(\frac{h}{\gamma m \mu}\right)\sqrt{\frac{1}{\lambda_0^2} + \frac{1}{\lambda'^2} - \frac{2\cos\theta}{\lambda_0\lambda'}}$$

$$\gamma\mu = \frac{h}{m}\sqrt{\frac{1}{\lambda_0^2} + \frac{1}{\lambda'^2} - \frac{2\cos\theta}{\lambda_0\lambda'}}$$

Let $\beta = \frac{\mu}{c}$

(6) $$\gamma\beta = \frac{h}{mc}\sqrt{\frac{1}{\lambda_0^2} + \frac{1}{\lambda'^2} - \frac{2\cos\theta}{\lambda_0\lambda'}} = b$$

Since $\gamma = \frac{1}{\sqrt{1-\beta^2}}$, from (6)

$$\frac{\beta}{\sqrt{1-\beta^2}} = b$$

$$\beta^2 = b^2(1-\beta^2) = b^2 - b^2\beta^2$$

$$b^2 = \beta^2 + b^2\beta^2$$

$$\beta = \frac{b}{\sqrt{1+b^2}}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-\frac{b^2}{1+b^2}}} = \frac{1}{\sqrt{\frac{1}{1+b^2}}} = \sqrt{1+b^2}$$

Isolate γ from (1)

$$\frac{hc}{mc^2}\left(\frac{1}{\lambda_0} - \frac{1}{\lambda'}\right) + 1 = \gamma = \sqrt{1+b^2}$$

$$\left(\left(\frac{hc}{mc^2}\right)\left(\frac{1}{\lambda_0} - \frac{1}{\lambda'}\right) + 1\right)^2 = 1 + b^2$$

$$\left(\frac{h}{mc}\right)^2\left(\frac{1}{\lambda_0^2} + \frac{1}{\lambda'^2} - \frac{2}{\lambda_0\lambda_1}\right) + \left(\frac{2h}{mc}\right)\left(\frac{1}{\lambda_0} - \frac{1}{\lambda'}\right) + 1 = 1 + b^2$$

From (6)

$$\left(\frac{h}{mc}\right)^2\left(\frac{1}{\lambda_0^2} + \frac{1}{\lambda'^2} - \frac{2}{\lambda_0\lambda'}\right) + \left(\frac{2h}{mc}\right)\left(\frac{1}{\lambda_0} - \frac{1}{\lambda'}\right) = \left(\frac{h}{mc}\right)^2\left(\frac{1}{\lambda_0^2} + \frac{1}{\lambda'^2} - \frac{2\cos\theta}{\lambda_0\lambda'}\right)$$

$$\left(\frac{h}{mc}\right)\left(\frac{-2}{\lambda_0\lambda'}\right) + 2\left(\frac{1}{\lambda_0} - \frac{1}{\lambda'}\right) = \left(\frac{h}{mc}\right)\left(\frac{-2\cos\theta}{\lambda_0\lambda'}\right)$$

$$\frac{1}{\lambda_0} - \frac{1}{\lambda'} = \frac{h}{mc}\left(\frac{1-\cos\theta}{\lambda_0\lambda'}\right)$$

$$(\lambda_0\lambda')\left(\frac{1}{\lambda_0} - \frac{1}{\lambda'}\right) = \frac{h}{mc}(1-\cos\theta)$$

$$\boxed{\lambda' - \lambda_0 = \frac{h}{m_e c}(1-\cos\theta)}$$

Q. E. D.

— Manoj Maddali
MSMS '09