$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-0.5}$$

$$f(x) = (1-x)^{-0.5} \qquad x = \frac{v^2}{c^2}$$

Taylor Series

$$f(x) = (1-x)^{-0.5} \qquad c = 0 \qquad n = \infty$$

$$f(x) = (1-x)^{-\frac{1}{2}} \qquad f(0) = 1$$

$$f'(x) = \frac{1}{2}(1-x)^{-\frac{3}{2}} \qquad f'(0) = \frac{1}{2}$$

$$f''(x) = \frac{1}{2}\left(\frac{3}{2}\right)(1-x)^{-\frac{5}{2}} \qquad f''(0) = \frac{3}{4}$$

$$f'''(x) = \frac{1}{2}\left(\frac{3}{2}\right)\left(\frac{5}{2}\right)(1-x)^{-\frac{7}{2}} \qquad f'''(0) = \frac{15}{8}$$

$$f''''(x) = \frac{1}{2}\left(\frac{3}{2}\right)\left(\frac{5}{2}\right)\left(\frac{7}{2}\right)(1-x)^{-\frac{9}{2}} \qquad f''''(0) = \frac{105}{16}$$

$$f^n(x) = \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdots (2n-1)}{2^n}(1-x)^{-\left(\frac{2n-1}{2}\right)} \qquad f^n(0) = \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdots (2n-1)}{2^n}$$

$$P_n = \left(\frac{1}{0!}\right)x^0 + \frac{1}{2}\left(\frac{1}{1!}\right)x + \frac{1}{2}\left(\frac{3}{2}\right)\left(\frac{1}{2!}\right)x^2 + \frac{1}{2}\left(\frac{3}{2}\right)\left(\frac{5}{2}\right)\left(\frac{1}{3!}\right)x^3 + \frac{1}{2}\left(\frac{3}{2}\right)\left(\frac{5}{2}\right)\left(\frac{7}{2}\right)\left(\frac{1}{4!}\right)x^4 + \dots + \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdots (2n-1)}{2^n\, n!}x^n$$

$$P_n = \gamma = 1 + \sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdots (2n-1)}{2^n\, n!}x^n$$

For $x \ll 1$

$$\boxed{\gamma \approx 1 + \frac{x}{2}}$$

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