Now use Cramer's Rule on Jarell's e.g.:

$y'' + y = \tan x \sec x$

Substitute $v_1(x)$ & $v_2(x)$ into y_h.

I. $\lambda^2 + 1 = 0$
$\lambda = \pm\sqrt{-1}$
$= \pm i$
II. $\cos x, \sin x$
III $y_h = c_1 \cos x + c_2 \sin x$

System: $v_1' \cos x + v_2' \sin x = 0$
$-v_1' \sin x + v_2' \cos x = q(x)$

$$v_1' = \frac{\begin{vmatrix} 0 & \sin x \\ q(x) & \cos x \end{vmatrix}}{\begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}} = \frac{0 - q(x)\sin x}{\cos^2 x + \sin^2 x}$$

$$= -\tan x \sec x \sin x$$

$$= -\tan^2 x$$

C.I.T. #63

$$v_1(x) = -\int \tan^2 x\,dx = -(-x + \tan x) + k_1$$
$$= -\tan x + x + k_1$$

$$v_2' = \frac{\begin{vmatrix} \cos x & 0 \\ \sin x & q(x) \end{vmatrix}}{\begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}} = \frac{q(x)\cos x - 0}{1}$$

$$= \tan x \sec x \cos x$$
$$= \tan x$$

C.I.T. #59

$$v_2(x) = \int \tan x\,dx = -\ln|\cos x| + k_2$$

$$y(x) = v_1(x)\cos x + v_2(x)\sin x$$
$$= (x - \tan x + k_1)\cos x + (-\ln|\cos x| + k_2)\sin x$$

$$= \underbrace{k_1 \cos x + k_2 \sin x}_{y_h} + \underbrace{(x - \tan x)\cos x + (-\ln|\cos x|)\sin x}_{y_p}$$

—W.F.

(p. 103)

#20 $\ddot{x} - 6\dot{x} + 9x = \frac{e^{3t}}{t^2}$ i.e. $\frac{d^2x}{dt^2} - 6\frac{dx}{dt} + 9x = e^{3t}t^{-2}$

I. $\lambda^2 - 6\lambda + 9 = 0$
$(\lambda-3)(\lambda-3) = 0$
$\lambda = 3, 3$ repeated R, CASE III

II. e^{3t}, te^{3t}

III. $c_1 e^{3t} + c_2 t e^{3t}$ Substitute $v_1(t)$ & $v_2(t)$ and apply Cramer's Rule.

System: $v_1' e^{3t} + v_2' t e^{3t} = 0$
$3v_1' e^{3t} + v_2'(e^{3t} + 3te^{3t}) = f(t)$

$$v_1' = \frac{\begin{vmatrix} 0 & te^{3t} \\ e^{3t}t^{-2} & (e^{3t}+3te^{3t}) \end{vmatrix}}{\begin{vmatrix} e^{3t} & te^{3t} \\ 3e^{3t} & (e^{3t}+3te^{3t}) \end{vmatrix}} = \frac{-t t^{-2} e^{6t}}{(e^{6t}+3te^{6t}) - (3te^{6t})} = \frac{-t^{-1}e^{6t}}{e^{6t}} = -\frac{1}{t}$$

$$v_1(t) = \int -\frac{dt}{t} = -\ln|t| + k_1$$

$$v_2' = \frac{\begin{vmatrix} e^{3t} & 0 \\ 3e^{3t} & e^{3t}t^{-2} \end{vmatrix}}{\text{same}} = \frac{t^{-2}e^{6t} - 0}{e^{6t}} = \frac{1}{t^2}$$

$$v_2(t) = \int t^{-2}\,dt = \frac{-1}{1}t^{-1} + k_2 = -\frac{1}{t} + k_2$$

Solution: $x(t) = v_1(t)e^{3t} + v_2(t)te^{3t}$

$$= (-\ln|t| + k_1)e^{3t} + \left(-\frac{1}{t} + k_2\right)te^{3t}$$

$$= k_1 e^{3t} + k_2 te^{3t} - (\ln|t|)e^{3t} - e^{3t}$$

$$\boxed{x(t) = (k_1 - 1)e^{3t} + k_2 te^{3t} - \ln|t|\, e^{3t}}$$

— W. F.