The Bohr Radius.

W. FUNMERBURK p.1

I. The EPE for the atom is

$$EPE = k_e \frac{(+e)(-e)}{r} = -k_e \frac{e^2}{r}$$

II. With the atom at rest, the TOTAL ENERGY is

$$E_{TOT} = KE_{(-e)} + EPE = \frac{1}{2} m_e v^2 - k_e \frac{e^2}{r}$$

By Newton's 2nd Law ($\Sigma F = ma$) and $\Sigma F = 0$

$$\Sigma F = m_e a_r - k_e \frac{e^2}{r^2} = 0$$

$$F_e = k_e \frac{e^2}{r^2} = m_e a_r = m_e \frac{v^2}{r}$$

$$k_e \frac{e^2}{r^2} = m_e \frac{v^2}{r}$$

Multiply both sides by $\frac{1}{2} r$

$$\frac{1}{2} r k_e \frac{e^2}{r^2} = \frac{1}{2} r m_e \frac{v^2}{r}$$

$$\frac{k_e e^2}{2r} = \frac{1}{2} m_e v^2 = KE_{(-e)}$$

So, the $KE_{(-e)} = \dfrac{k_e e^2}{2r}$

III. Thus if $E_{TOT} = KE_{(-e)} + EPE = \frac{1}{2} m_e v^2 - k_e \frac{e^2}{r}$, then

$$E_{TOT} = k_e \frac{e^2}{2r} - k_e \frac{e^2}{r}$$

$$= k_e \frac{e^2}{r}\left(\frac{1}{2} - 1\right)$$

$$= -k_e \frac{e^2}{2r}$$

The negative sign indicates the $(-e)$ is bound to the $(+e)$

IV. Use Bohr's assumption $L = m_e v r = n\frac{h}{2\pi} = n\hbar$,

and $F_e = k_e \frac{e^2}{r^2} = m_e \frac{v^2}{r}$ (i.e., $\Sigma F = 0$),

and then solve for v and then r.

Here goes...

A. $\begin{cases} k_e \frac{e^2}{r^2} = m_e \frac{v^2}{r} \\ v^2 = \frac{k_e}{m_e} \frac{r e^2}{r^2} \\ v^2 = \frac{k_e}{m_e} \frac{e^2}{r} \end{cases}$ B. $\begin{cases} L = m_e v r = n\hbar \\ v = \frac{n\hbar}{m_e r} \\ v^2 = \frac{n^2\hbar^2}{m_e^2 r^2} \end{cases}$

$$v^2 = \frac{k_e}{m_e} \frac{e^2}{r} = \frac{n^2 \hbar^2}{m_e^2 r^2}$$

and solve for r.

$$(m_e r)\left[\frac{k_e e^2}{m_e r}\right] = (m_e r) \frac{n^2 \hbar^2}{m_e^2 r^2}$$

$$\frac{k_e e^2}{1} = \frac{n^2 \hbar^2}{m_e r}$$

$$m_e r k_e e^2 = n^2 \hbar^2$$

$$r_n = \frac{n^2 \hbar^2}{m_e k_e e^2} \quad \forall\, n = 1, 2, 3 \ldots \in \mathbb{N}.$$

This is the Bohr Radius.

$$r_1 = \frac{(1)^2 (\frac{1}{2\pi})^2 (6.63 \times 10^{-34}\,J \cdot s)^2}{(9.11 \times 10^{-31}\,kg)(8.99 \times 10^{9}\,\frac{N \cdot m^2}{C^2})(1.60 \times 10^{-19}\,C)^2}$$

$= 5.3 \times 10^{-11}\,m$

$= 0.53\,\text{Å}$

— W.F.

Energies of the Quantum States of the Bohr Atom

I. $v^2 = \frac{n^2 \hbar^2}{m_e^2 r^2} = \frac{k_e e^2}{m_e r}$ (See p. 2) W. FUNDERBURK ph

and $r_n = \frac{n^2 \hbar^2}{m_e k_e e^2}$ } Plug it in.

$$v^2 = \frac{k_e e^2}{\cancel{m_e}} \cdot \frac{m_e k_e e^2}{n^2 \hbar^2} = \left(\frac{1}{n^2}\right)\frac{k_e^2 e^4}{\hbar^2}$$

II. Multiply then by $\frac{1}{2} m_e$ to get $KE_{(-e)}$

$$KE_n = \left(\frac{1}{n^2}\right) \frac{m_e k_e^2 e^2}{2\hbar^2} \quad \forall\, n = 1, 2, 3, \ldots \in \mathbb{N}.$$

$$KE_{n=1} = \frac{(9.11\times10^{-31}\,kg)(8.99\times10^{9}\,\frac{Nm^2}{C^2})^2(1.60\times10^{-19}\,C)^4}{(2)(6.63\times10^{-34}\,J.s)^2}$$

$$= 2.14\times10^{18}\,J \times \frac{1\,eV}{1.60\times10^{-19}\,J} \approx \underline{13.6\,eV}$$

III. $E_{IONIZATION} = -E_{BINDING} = -13.6\,eV$ for $n=1$

The negative sign means the $(-e)$ is bound to the $(+e)$.

$$E_n = -\frac{13.6}{n^2}\,eV$$

PHOTON EMISSION (FROM BOHR HYDROGEN ATOM)

W. FUNKENBURG p.4

I. Because $E = hf$,

$$f = \frac{\Delta E}{h} = \frac{E_o - E_f}{h}$$

$$= \frac{1}{h}\left[\left(\frac{1}{n_o^2}\right)\frac{m_e k_e^2 e^4}{2\hbar} - \left(\frac{1}{n_f^2}\right)\frac{m_e k_e^2 e^4}{2\hbar}\right]$$

$$= \frac{2\pi}{2\pi h}\,\frac{m_e k_e^2 e^4}{2\hbar^2}\left(\frac{1}{n_o^2} - \frac{1}{n_f^2}\right)$$

$$= \frac{m_e k_e^2 e^4}{4\pi\hbar^3}\left(\frac{1}{n_o^2} - \frac{1}{n_f^2}\right)$$

II. $c = f \cdot \lambda$

$$\frac{1}{\lambda} = \frac{f}{c} = \underbrace{\frac{m_e k_e^2 e^4}{4\pi \cdot c \cdot \hbar^3}}_{R_H}\left(\frac{1}{n_f^2} - \frac{1}{n_o^2}\right)$$

III. $$R_H = \frac{(9.11\times10^{-31}\,kg)(8.99\times10^{9}\,\frac{Nm^2}{C^2})^2(1.60\times10^{-19}\,C)^4}{(4\pi)(2.99\times10^{8}\,\frac{m}{s})(\frac{1}{2\pi})^3(6.63\times10^{-34}\,J\cdot s)^3}$$

$$\approx 1.10\times10^{7}\,m^{-1}$$

but with more precision $R_H = 1.097\times10^{7}\,m^{-1}$

IV. For $n_f = 2$ (i.e., BALMER SERIES)

$$\frac{1}{\lambda} = R_H\left(\frac{1}{2^2} - \frac{1}{n_o^2}\right) \quad \text{or} \quad \lambda = \frac{4}{R_H}\left[\frac{(n_o)^2}{(n_o)^2 - 4}\right]$$

— W.F.